FOUR PRECIOUS JEWELS

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I will talk about some aspects of four remarkable objects: the countable random graph (or Rado graph), the rational numbers (as ordered set), the Urysohn metric space, and the pseudo-arc. These objects appear in many different areas of mathematics, but they have certain features in common: they can be constructed as suitable limits of finite combinatorial structures; many of their properties can be investigated by combinatorial techniques; and they provide mechanisms for some very interesting interactions between combinatorics and other parts of mathematics (for example, the KPT theorem connecting Ramsey theory with topological dynamics).

ON DECOMPOSING REGULAR GRAPHS AND MULTIGRAPHS INTO ISOMORPHIC TREES AND FORESTS

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Let $H$ and $G$ be graphs or multigraphs such that $G$ is a subgraph of $H$. A $G$-decomposition of $H$ is a set $\Delta = \{G_1, G_2, \ldots, G_t\}$ of pairwise edge-disjoint subgraphs of $H$ each of which is isomorphic to $G$ and such that each edge of $H$ occurs in exactly one $G_i$. Graham and Häggkvist have conjectured that every tree with $n$ edges decomposes every $2n$-regular graph as well as every $n$-regular bipartite graph. These conjectures have been confirmed for a small number of cases. We believe the Graham and Häggkvist Conjectures extend to forests with $n$ edges. We have also recently conjectured that every tree with $n$ edges decomposes every $2n$-regular multigraph with edge multiplicity at most 2. In this talk, we report on some recent results related to variations of these conjectures.
GROUP DIVISIBLE COVERINGS AND RELATED STRUCTURES

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Group divisible coverings, GDCs, are a covering generalization of group divisible designs. More formally, a \((t, k)\)-GDC of type \(g^u\) is a triple \((V, \mathcal{G}, \mathcal{B})\), where \(V\) is a set of \(gu\) points, \(\mathcal{G}\) is a partition of \(V\) into \(u\) subsets of size \(g\), called groups, and \(\mathcal{B}\) is a collection of \(k\) subsets of \(V\), called blocks, such that (1) no pair of points belonging to the same group is contained in a block, (2) every set of \(t\) points which intersects \(t\) distinct groups is contained in at least one block.

Group divisible coverings are related to many well-studied families of combinatorial designs. For example, group divisible designs, orthogonal and covering arrays, BIBDs, projective planes, \(t\)-designs, and coverings are equivalent to specific families of GDCs. Note that the block size of these well-known combinatorial structures varies in relation to the number of the groups (or points).

Here, we consider the block size \(k\) of GDCs to be an integer function of the number of groups \(g\), \(k = k(g)\). We observe how the size and nature of GDCs changes when \(k\) has different orders of magnitude compared to \(g\) and how this impacts the approach to studying GDCs and the related structures.

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SAMPLEING AND COUNTING USING MARKOV CHAINS

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A Markov chain is a simple stochastic process which is “memoryless”: the next state depends only on the current state, not on the entire history. Classical Markov chain theory investigated conditions under which the distribution of the current state of the chain would converge to a unique stationary distribution. About 30 years ago, motivated by applications in statistical physics and computer science, a new question emerged: how quickly does the Markov chain tend to its unique stationary distribution, as a function of the size of the state space?

I will aim to give a flavour of some of the breakthrough results in the use of Markov chains for sampling and counting, the methods which have been used to prove them, and connections with statistical physics.
**MATCHINGS IN TRIPARTITE HYPERGRAPHS**

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A matching in a hypergraph is a set of disjoint edges. It is a well-known difficult problem to give good lower bounds on the maximum size of a matching in a hypergraph in terms of other natural parameters. Here we focus on the special case of tripartite hypergraphs: those for which the vertex set can be partitioned into three parts, such that each edge contains exactly one vertex from each part. If a tripartite hypergraph is $r$-regular (meaning that each vertex is in exactly $r$ edges) with $n$ vertices in each class then it has a matching of size at least $n/2$, and this is tight for certain special hypergraphs. We investigate how this bound can be improved for all other hypergraphs.

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**2048 IDEAS FOR TURNING COMBINATORIAL RESEARCH INTO A GAME**

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What could be achieved if combinatorial research were as enticing and addictive as playing a great computer game like 2048? What are the key design attributes of such a “research game”? I’ll explore these questions in light of some recent joint research projects with students, exposing aspects of the research process that are usually omitted from the final published version. Prior addiction to 2048 will not be assumed.
A homomorphism from a graph $G$ to a graph $H$ is a mapping $\varphi : V(G) \to V(H)$, not necessarily injective, that maps edges of $G$ to edges of $H$. Homomorphisms into complete graphs are the same as graph colourings, and many colouring-type parameters of graphs can be defined as the existence of homomorphisms into other specific families of graphs. Homomorphic equivalence (that is, the existence of homomorphisms in both directions between two graphs) is an equivalence relation, and each equivalence class contains a unique smallest graph, called a core (or the core of any of the graphs in the equivalence class). If a graph is highly structured, then the core often inherits some of this structure - for example, cores of vertex-transitive graphs are vertex-transitive, but the full extent to which “vertex-transitive can be replaced by other properties is not known. In this talk, I will discuss a number of results and problems related to graph homomorphisms, endomorphisms (i.e., homomorphisms from a graph to itself) and cores.
Graph products and covering arrays

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Covering arrays are relaxation of orthogonal arrays to covering hypothesis that have been successfully applied in the design of test suites for testing systems such as software, circuits and networks, where failures can be caused by the interaction between their parameters. Two vectors \( x, y \) in \( \mathbb{Z}_g^n \) are qualitatively independent if for all pairs \((a, b) \in \mathbb{Z}_g \times \mathbb{Z}_g\), there exists \( i \in \{1, 2, \ldots, n\} \) such that \((x_i, y_i) = (a, b)\). A covering array on a graph \( G \), denoted by \( CA(n, G, g) \), is a \(|V(G)| \times n\) array on \( \mathbb{Z}_g \) with the property that any two rows which correspond to adjacent vertices in \( G \) are qualitatively independent. The number of columns in such array is called its size. Given a graph \( G \), a covering array on \( G \) with minimum size is called optimal. We consider four most extensively studied graph products in literature and give upper and lower bounds on the size of covering arrays on these product graphs. Using automorphism group of factor graph, we give some classes of cartesian product graphs for which the size of covering arrays on cartesian product graphs achieve the lower bound.

Combinatorial results for the semigroup of order-preserving contraction mappings of a finite chain

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(Joint work with Goerg Kelin, Nadia Al Dhamri, Abdullahi Umar)

Let \( I_n \) be the partial one-to-one symmetric semigroup on \( X_n = \{1, 2, \ldots, n\} \) and let \( OCI_n, ODCI_n \) be its subsemigroups of order-preserving partial one-to-one contraction and subsemigroup of order-preserving and order decreasing partial one-to-one contraction mappings of \( X_n \) respectively. In this talk we investigate the cardinalities of some equivalences on \( OCI_n \) and \( ODCI_n \).
THE EDGE SLIDE GRAPH OF A STRICTLY REDUCIBLE SIGNATURE OF THE $n$-DIMENSIONAL CUBE

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An edge slide occurs in a spanning tree $T$ of the $n$-dimensional cube $Q_n$ when an edge of $T$ can be slid across a 2-dimensional face to produce another spanning tree. The edge slide graph is the graph whose vertices are the spanning trees of $Q_n$, with an edge between two trees if they are related by an edge slide. The signature of a spanning tree of $Q_n$ is the $n$-tuple $(a_1, \ldots, a_n)$, where $a_i$ is the number of edges in the $i$th direction. A signature of $Q_n$ is reducible if after being permuted to increasing order it has an initial segment which is a signature of a lower dimensional cube. A signature is strictly reducible if the signature of the lower cube is not followed by a sequence of powers of two. Signatures are invariant under edge slides, so trees with different signatures belong to different components of the edge slide graph. Our conjecture is that the edge slide graph of a signature of $Q_n$ is disconnected if and only if the signature is strictly reducible. This presentation outlines how we characterised the class of $n$-tuples that are signatures of a spanning tree, and discuss the proof of one direction of the conjecture.

PATTERN MATCHING IN UNIONS OF TWO MONOTONE SEQUENCES

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(Joint work with Marie-Louise Bruner, Martin Lackner and Vincent Vatter)

There is a large body of research dealing with finding efficient algorithms for determining whether or not a pattern occurs in a text, depending of course on precisely what one means by the terms ‘pattern’ and ‘text’. One interpretation is where patterns and texts are permutations, and an occurrence of a pattern $\pi$ in a text $\tau$ is a subsequence of $\tau$ whose elements are in the same relative order as all the elements of $\pi$. In this context the general problem with parameters $k$, the size of the pattern, and $n$, the size of the text, is known to be NP-complete. Recently, Guillemot and Marx demonstrated an algorithm of complexity $2^{O(k^2 \log k)}n$, thereby showing that the problem is fixed-parameter tractable. We consider a restricted case where both pattern and text can be written as the union of two monotone sequences and illustrate simple $O(kn)$ algorithms in this context.
A NEW VERSION OF THE HIT-AND-RUN ALGORITHM TO SAMPLE GRAPH SPACES

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(Joint work with A. K. Azad and Jonathan M. Keith)

We propose a Markov Chain Monte Carlo (MCMC) sampler as a new approach to sample graph spaces from a discrete distribution \( f(N, E | D) \) that is defined on a finite graph space \( X \), where \( D \) represents data-points observed at discrete times, \( N \) is a set of vertices representing variables, and \( E \) is a set of directed edges describing the causal relationships between variables. The new sampling technique aims to substantially resolve the local maximum problem that arises in sampling graph spaces. The new sampler is related to the Hit-and-Run (HAR) sampler (see Smith 1984). The original HAR has been shown to speedily converge to the target distribution with a low probability of getting “stuck” at a local maximum (see Gilks and Richardson 1996, Lovasz 1999, Lovasz 2003). The new approach preserves many of the attributes of the original HAR sampler but is accommodated, in this study, to meet the problem of generating graphs from a discrete distribution. Consistent with the original HAR sampler, at iteration \( t \), we formulated the linear equation: \( g_{t+1} = \ell_t p_t + g_t \), where \( g_t \) and \( g_{t+1} \) are the current graph and the next iterated graph, respectively, \( p \) is a random path that represents a sequence of graphs and \( \ell_t \) is the length of \( p_t \), that is, \( |p| \geq 1 \). The term \( \ell_t, p_t \) facilitates large movements across the graph-space, which in principle should enable good mixing and rapid convergence. In this work, both \( \ell_t, p_t \) are iteratively sampled uniformly over some interval, such that every single graph in the space is accessible from the current graph in one iteration with some positive probability. Primary results demonstrate the sampler greatly alleviates the problems caused by local maxima, which in turn facilitates exploring the entire posterior distribution.

HYPERGRAPH COLORING UP TO CONDENSATION

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A hypergraph is \( k \)-uniform if every hyperedge contains \( k \) vertices. A colouring of a hypergraph is an assignment of colours to the vertices such that no hyperedge is monochromatic. We consider the problem of \( q \)-colouring a random \( k \)-uniform hypergraph with \( n \) vertices and \( cn \) edges, where \( q, k \) and \( c \) are constants and \( n \) tends to infinity. Most recently Dyer, Frieze and Greenhill (2014) determined the \( q \)-colourability threshold up to a multiplicative \( 1 + o(1) \) factor. However, due to changes in the geometry of the solution space a vanilla second moment argument becomes untenable at edge densities above those considered. Following developments by Coja-Oghlan and Vilenchik (2014) in graph \( q \)-colouring, we are able to reduce the uncertainty in the threshold to an additive error of \( \ln 2 + o(1) \). In order to do so, we consider a new variable which is informed by non-rigorous physics predictions on the geometry of the solution space of colours. This new threshold matches the conjectured location of the 'condensation phase transition' which poses yet another technical barrier.
We consider designs where each block is a circle, or can be considered as such by adjoining border plots. Sometimes there is the extra complication that plots are cross-classified in a rectangle where columns are blocks and the rows are also important.

The design is neighbour-balanced if (i) no treatment follows itself and (ii) every treatment follows every other treatment equally often. Such designs require a large number of plots. Weak neighbour balance, which can often be achieved in fewer plots, replaces (ii) by a combinatorial condition on the incidence matrix for treatments following each other. Familiar combinatorial objects such as doubly regular tournaments, 2-designs, strongly regular graphs and \( S \)-digraphs can be used to construct circular designs with weak neighbour balance.

In 1967, F. Arthur Sherk gave a simple proof that the finite metric planes (of Bachmann and Schmidt) are precisely the affine planes of odd order. Moreover, Sherk’s proof holds for a more general class of incidence structures that do not involve the ‘three-reflection theorem’ whatsoever, and thus yields a beautiful characterisation of the finite affine planes of odd order. By relaxing the first of Sherk’s axioms to ‘every pair of points lies on at most one line’, we can study what we call partial Sherk planes. In this talk, we outline our characterisation of these incidence structures as Bruck nets, in the same vein as Sherk’s result, and what it means for connected combinatorial objects such as mutually orthogonal latin squares.
TRIANGLE DECOMPOSITIONS OF ALMOST-COMPLETE GRAPHS

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(Joint work with Kelly Pham, Landon Settle, Kayla Wright)

A triangle decomposition of a graph $G$ is a partition of the edges of $G$ into triangles. Two simple conditions needed for $G$ to admit such a decomposition are that $|E(G)|$ is a multiple of three and that the degree of any vertex in $G$ is even; we call such graphs tridivisible.

Not all tridivisible graphs admit triangle decompositions; consider a hexagon. For complete graphs, however, this “obviously necessary” condition turns out to be sufficient. This raises a natural question: for what other families of graphs does tridivisibility ensure the existence of a triangle decomposition?

Nash-Williams conjectured that these necessary conditions would be sufficient for almost-complete graphs, which for this talk’s purposes we interpret as any graph $G$ on $n$ vertices with $\delta(G) \geq (1 - \epsilon)n$, $E(G) \geq (1 - \xi)(\binom{n}{2})$ for some appropriately small constants $\epsilon$, $\xi$. In this talk we discuss recent results on this conjecture.

FINDING LONG TRANSVERSALS IN LATIN SQUARES

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(Joint work with Ian Wanless)

A partial transversal of a latin square of order $n$ is a subset of entries picked in such a way that each row, each column and each symbol is present at most once. In many latin squares, you can find a full transversal by selecting $n$ entries which do not duplicate any row, column or symbol. But what about when you can’t find a full transversal? Brualdi has conjectured that a partial transversal of length at least $n - 1$ is always present in any latin square. In this talk, we will discuss recent work which shows that for small orders, Brualdi’s conjecture holds. We show that his conjecture also holds for small generalized latin squares.
Maximum degree and diameter-bounded subgraph (MaxDDDBS) is a generalisation of the degree diameter problem (DDP) in which the aim is to maximise the number of vertices in a graph with given degree and diameter. In DDP, we can add any number of vertices and edges to a graph as long as we satisfy the degree and diameter constraints. However, in MaxDDDBS, our selection of edges is restricted by the requirement that the resulting graph is a subgraph of some given host architecture. In this talk, we will discuss the MaxDDDBS problem on Benes network of given maximum degree 4 which is the maximum degree of the network. Furthermore, we will discuss the lower bound for subgraphs of various diameters when we restrict the maximum degree to 3 and 2.

We analyze asymptotic expansions of the binomial coefficients and the Catalan numbers and obtain some new interesting formulas introducing shifted variable in the asymptotic expansion. Asymptotic formulas for the sum of the binomial coefficients and the Catalan numbers are also derived and presented.
THE HARDNESS OF SIMPLIFYING TRIANGULATIONS

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(Joint work with Thomas Lewiner, Joao Paixao and Jonathan Spreer)

In many topological algorithms, input is given as a triangulation—a combinatorial representation of a topological space. The set of all triangulations of a given space can be modelled as an infinite graph, whose vertices represent triangulations, and whose edges represent local modifications.

Exploring this graph, and in particular finding a path to a smaller triangulation, plays an essential and extremely effective role in topological software. Nevertheless, we show here that some of the underlying problems are (assuming P \(\neq\) NP) intrinsically hard. For instance, we show that even for the simple case of a triangulated 3-sphere, it is NP-complete to find a path of length \(k\) that monotonically simplifies the input.

We also discuss links with parameterised complexity and kernelisation, as well as implications for practical software.

ORTHOMORPHISMS IN THE CYCLIC GROUP AND TRADES IN MOLS

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(Joint work with Diane Donovan and Fatih Dermikale)

If \(n\) is odd, the function \(\theta(x) = 2x\) is trivially an orthomorphism of the cyclic group of order \(n\); that is \(\theta\) is a permutation of the set \(\{0, 1, \ldots, n-1\}\) such that \(x \rightarrow \theta(x) - x\) is also a permutation of this set. We show that if \(\theta' \neq \theta\) is also an orthomorphism of the cyclic group, \(\theta'\) differs from \(\theta\) in \(\Omega(\log{n})\) places. Moreover, we also show that for each \(n\) there exists such a \(\theta'\) differing from \(\theta\) in \(O(\log{n})\) places. We arrived at this research question by studying trades within pairs of MOLS from a set of \(n-1\) MOLS of order \(n\) when \(n\) is prime.
Finite vertex-transitive graphs may be classified according to the action of the automorphism group on the arcs (ordered edges) of the graph.

Let $X$ be a vertex-transitive graph of valency $d$, with full automorphism group $A$. Then the arc-type of $X$ is defined in terms of the lengths of the orbits of the action of the stabiliser $A_v$ of a given vertex $v$ on the set of arcs emanating from $v$. Specifically, the arc-type is the partition of $d$ as the sum

$$n_1 + n_2 + \cdots + n_t + (m_1 + m_1) + (m_2 + m_2) + \cdots + (m_s + m_s),$$

where $n_1, n_2, \ldots, n_t$ are the lengths of the self-paired orbits, and $m_1, m_1, m_2, m_2, \ldots, m_s, m_s$ are the lengths of the non-self-paired orbits. This is a graph invariant. For example, if $X$ is arc-transitive then its arc-type is $d$, while if $X$ is half-arc-transitive then its arc-type is $(d/2 + d/2)$, and if $X$ is zero-symmetric (or equivalently a graphical regular representation (GRR) of the group $A$), then $n_i = 1$ and $m_j = 1$ for all $i$ and $j$.

In this talk I will explain how it can be shown using Cartesian products of ‘relatively prime’ examples of certain kinds of VT graphs that every partition of the given form occurs as the arc-type of some vertex-transitive graph, with the exception of $1 + 1$ and $(1 + 1)$. This will be taken further by my research student Nemanja Poznanovic in his talk, showing that every such partition is the arc-type of some Cayley graph.
DISPLAYING AND COUNTING TREES IN A PHYLOGENETIC NETWORK

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A phylogenetic network is a generalized phylogenetic tree (evolutionary tree). The main difference between a phylogenetic network and a phylogenetic tree is the presence of reticulation vertices in the former. A reticulation vertex has two directed edges going into it and one directed edge going out, whereas all other vertices (apart from the root and leaves) have one directed edge going in and two directed edges going out. If exactly one incoming edge is deleted for each reticulation vertex in a phylogenetic network then, after some tidying up, a phylogenetic tree on the same leaf set as the phylogenetic network is obtained (displayed). Since there are two incoming edges for each reticulation vertex, there is a maximum of $2^k$ distinct trees that can be obtained from a phylogenetic network with $k$ reticulation vertices.

This talk explores the following questions: Given a positive integer $n$, does there exist a phylogenetic network $N$ such that exactly $n$ distinct trees are obtained from $N$? Is there a non-trivial class of phylogenetic networks such that counting the number of trees displayed by any network in the class is polynomial-time?

COORDINATISING PROJECTIVE PLANES USING FINITE FIELDS

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We revisit the method of coordinatising projective planes, which produces a planar ternary ring (PTR), a three-variabed function defined on the labelling set, containing the algebraic properties of the plane. For planes of prime power order $q$, the finite field $F_q$ can be used as the labelling set. This leads to the concept of a PTR polynomial. Restrictions on the general form of a PTR polynomial are then derived from the properties a PTR function must exhibit. With further assumptions on the plane, further restrictions on the PTR polynomial can be derived.
**THE CHALLENGE OF FINDING "RANDOM" MATROIDS**

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In the field of graph theory there is an obvious, intuitive way to define (and generate) "random graphs" of type $G(n, p)$ and these objects have been studied in great detail. Matroids in contrast yield no such straightforward means of generating randomness, which prevents us from using probabilistic methods to establish the existence or prevalence of certain properties amongst matroids. However progress towards this ideal may be possible if we restrict to certain significant classes of matroids, particularly if also fixing the rank of the matroid. This talk will introduce some results for these matroids, in particular for the class of transversal matroids, and discuss other possibilities in this area.

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**ON THE HAMILTON DECOMPOSABILITY OF VERTEX-TRANSITIVE MULTIGRAPHs**

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(Joint work with Darryn Bryant)

Let $kG$ denote the multi-graph obtained from graph $G$ by replacing each edge of $G$ with an edge of multiplicity $k$. We explore the question:

For which vertex-transitive graphs, and which values of $k$ is $kG$ decomposable into Hamilton cycles (and possibly a 1-factor)?

In particular, if we consider graphs of order up to 20 vertices: the case $k = 1$ of this question finds only two graphs: $P$ and $L(P)$, which are not Hamilton decomposable, but for the case $k > 1$, there are twenty more small graphs which have at least one multiple $kG$ without a Hamilton decomposition.
LATIN HYPERCUBE SAMPLING AND THE AGE OF GROUND WATER

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In this talk I will discuss how Latin Hypercube sampling can be used to quantify uncertainty in problems such as determining the age of ground water. I will give an outline of how combinatorial and analytic arguments can be used to prove that the percentage coverage after performing \( k \) trial samples, in \( d \) dimensional parameter space of size \( n \), takes the form

\[
P(k, n, d, t) = 1 - e^{-k/n^t-1}.
\]

Bibliography


THE HISTORY OF TUTTE-WHITNEY POLYNOMIALS

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The Tutte-Whitney polynomial of a graph is a two-variable polynomial that contains a lot of interesting information about the graph. It includes, for example, the chromatic, flow and reliability polynomials of a graph, the Ising and Potts model partition functions of statistical mechanics, the weight enumerator of a linear code, and the Jones polynomial of an alternating link. Specific evaluations can be used to count colourings, flows, acyclic orientations, spanning trees, spanning subgraphs, and forests. It plays a central role in enumerative graph theory, and has been extended to matroids, polymatroids, greedoids, maps, and boolean functions, among other structures.

We describe the early history of Tutte-Whitney polynomials, especially the contributions of the early papers of Whitney and Tutte.
ARCTRANSITIVE DIGRAPHS WITH QUASIPRIMITIVE VERTEX STABILIZERS

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(Joint work with Michael Giudici, Cai Heng Li, Gabriel Verret)

Let \( \Gamma \) be a connected digraph, and suppose that \( G \leq \text{Aut}(\Gamma) \) is transitive on the arcs of \( \Gamma \). Given a vertex \( v \), suppose that the stabilizer \( G_v \) acts quasiprimiately on the in-neighbours of \( v \), and the out-neighbours of \( v \). We prove that the types of actions are severely constrained.

PERCOLATION ON RANDOM DIRECTED GRAPHS

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(Joint work with Jane Gao)

Let \( G \) be a random directed graph with given in- and out-degree sequence and \( p \in [0, 1] \). Delete each vertex (together with all incident edges) with probability \( 1 - p \), independently of the other vertices. We denote the resulting random graph by \( G_p \).

In this talk, we present a threshold for the presence of a giant strongly connected component in \( G_p \). Specifically, we determine \( p_c \) such that for \( p > p_c \), \( G_p \) asymptotically almost surely contains a strongly connected component of size \( \Theta(n) \) and for \( p < p_c \), asymptotically almost surely every strongly connected component of \( G_p \) has size \( O(\Delta^2 \log(n)) \) where \( \Delta \) is the minimum of the maximum in-degree and maximum out-degree.
ON THE PARTITION DIMENSION OF CIRCULANT GRAPHS

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(Joint work with Sudeep Stephen, Bharati Rajan, Mirka Miller, Joe Ryan)

For a vertex $v$ of a connected graph $G(V,E)$ and a subset $S$ of $V$, the distance between $v$ and $S$ is defined by $d(v,S) = \min\{d(v,x) : x \in S\}$. For an ordered $k$-partition $\Pi = \{S_1, S_2, \ldots, S_k\}$ of $V$, the representation of $v$ with respect to $\Pi$ is the $k$-vector $r(v|\Pi) = (d(v,S_1), d(v,S_2), \ldots, d(v,S_k))$. The $k$-partition $\Pi$ is a resolving partition if the $k$-vectors $r(v|\Pi), v \in V$ are distinct. The minimum $k$ for which there is a resolving $k$-partition of $V$ is the partition dimension of $G$. In this paper we obtain the partition dimension of circulant graphs $G = C(n, \pm\{1,2,\ldots,j\}), 1 \leq j < \lfloor \frac{n}{2} \rfloor$, $n \geq (j+k)(j+1)$.

SKOLEM CIRCLES

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A Skolem sequence on $m$ symbols is a sequence of $2m$ symbols from the symbol set $\{1,2,\ldots,m\}$ with each symbol appearing twice, such that the distance between any pair of that same symbol is that symbol value in integers. For example $41134232$ is a Skolem sequence on 4 symbols. Skolem sequences have various cousin sequences and generalisations such as hooked Skolem sequences, $k$-fold Skolem sequences, Langford sequences and Rosa Sequences.

We investigate wrapping a Skolem sequence around a circle. A Skolem circle is a circular configuration of $2m$ cells which can be filled with the symbol set $\{1,2,\ldots,m\}$ with each symbol appearing twice, such that the distance between any pair of that same symbol is that symbols value in integers. There are some Skolem circles which cannot be unwrapped into a Skolem sequence.
CYCLES OF A GIVEN SIZE IN A DENSE GRAPH

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(Joint work with David R. Wood)

A well-known result of Corrádi and Hajnal states that every graph with at least $3k$ vertices and minimum degree at least $2k$ contains $k$ vertex disjoint cycles. Recently, Wang showed that, when $k \geq 2$, every graph with at least $4k$ vertices and minimum degree at least $2k$ either contains $k$ vertex disjoint cycles of length at least 4, or is one of a small set of exceptions. We prove a similar result for vertex disjoint cycles of length at least $r$, in terms of average degree instead of minimum degree: every graph with average degree at least $\frac{4}{3}kr$ contains $k$ disjoint cycles of length at least $r$, as long as $k \geq 6$.

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2-NEIGHBOUR TRANSITIVE CODES

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(Joint work with Neil Gillespie, Michael Giudici and Cheryl Praeger)

Combinatorial symmetry has often been an important consideration in the study of error-correcting codes, such as perfect codes and completely regular codes. Though some classification results have been achieved in this direction, a full classification is still quite far away. Here we discuss a related algebraic symmetry condition.

In this talk, an $(X,2)$-neighbour transitive code is a subset of the vertices of a Hamming graph, such that the following three sets are $X$-orbits: the code itself, the vertices distance 1 from the code, and vertices distance 2 from the code. We discuss progress towards a classification of $(X,2)$-neighbour transitive codes with minimum distance $\delta \geq 5$. 
THE EXTREMAL FUNCTION FOR PETERSEN MINORS

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(Joint work with David Wood)

A graph $H$ is a minor of a graph $G$ if $H$ can be obtained from a subgraph of $G$ by contracting edges. A natural question at the intersection of graph minor theory and extremal graph theory is: given a graph $H$, what is the maximum number of edges in an $n$-vertex graph with no $H$ minor? We answer this question when $H$ is the Petersen graph, and show how this result leads to a tight bound on the chromatic number of Petersen minor free graphs. No knowledge of graph minor theory is assumed.

RYSER’S CONJECTURE FOR LINEAR INTERSECTING HYPERGRAPHS

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(Joint work with Nevena Francetić, Brendan McKay, Ian Wanless)

A hypergraph is $r$-partite if its vertex set can be partitioned into $r$ sets such that every line consists of exactly one vertex from each set. A long-standing conjecture by Ryser says that every $r$-partite hypergraph with covering number $\tau$ and matching number $\nu$ satisfies $\tau \leq (r - 1)\nu$. However, the conjecture is only known upto $r = 3$. In this talk I will outline a proof of Ryser’s Conjecture for $r \leq 9$ in the special case of linear intersecting hypergraphs, which have the property that every pair of lines meet in exactly one vertex.
Locating arrays are combinatorial objects used for designing testing procedures that identify and locate faults. It turns out that the existence problem for a simple kind of locating array is equivalent to a natural question in extremal set theory concerning families of disjoint partitions. In this talk I will introduce locating arrays, discuss this equivalence, and outline a complete solution to the problem.

Decomposing the complete graph with a hole into short cycles

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(Joint work with Daniel Horsley)

The complete graph of order \( v \) with a hole of size \( u \), denoted \( K_v - K_u \), is the graph obtained from a complete graph of order \( v \) by removing the edges of a complete subgraph of order \( u \). In 2014 Bryant, Horsley and Pettersson proved that the complete graph can be decomposed into cycles of arbitrary lengths, provided that the obvious necessary conditions hold. In this talk we present some results for decomposing \( K_v - K_u \) into cycles of arbitrary lengths. We also outline the necessary conditions for the existence of these decompositions. The main focus of the talk will be results for packing cycles of length at most 6 into \( K_v - K_u \). These packings are useful for obtaining decompositions of \( K_v - K_u \) into cycles of longer lengths.
ON THE METRIC DIMENSION OF WHEEL RELATED GRAPHS

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(Joint work with Syed Ahtsham Ul Haq Bokhary, Zill-E-Shams)

If $G$ is a connected graph, the distance $d(u,v)$ between two vertices $u,v \in V(G)$ is the length of a shortest path between them. Let $W = \{w_1, w_2, \ldots, w_k\}$ be an ordered set of vertices of $G$ and let $v$ be a vertex of $G$. The representation $r(v|W)$ of $v$ with respect to $W$ is the $k$-tuple $(d(v,w_1), d(v,w_2), \ldots, d(v,w_k))$. $W$ is called a resolving set or a locating set if every vertex of $G$ is uniquely identified by its distances from the vertices of $W$, or equivalently, if distinct vertices of $G$ have distinct representations with respect to $W$. A resolving set of minimum cardinality is called a metric basis for $G$ and this cardinality is the metric dimension of $G$, denoted by $\beta(G)$. Metric dimension is a generalization of affine dimension to arbitrary metric spaces (provided a resolving set exists).

The metric dimension of some wheel related graphs are studied recently in [H. M. A. Siddiqui, M. Imran, Appl. Math. Comput. 244(2014), 624 – 632]. In this paper, we study the metric dimension of wheels with $k$ consecutive missing spokes denoted by $W_{n,k}$. We compute the exact value of metric dimension of $W_{n,k}$ which shows that wheels with consecutive missing spokes have unbounded metric dimension. It is natural to ask for characterization of graphs with unbounded metric dimension. The exchange property for resolving set of $W_{n,k}$ has also been studied in this paper and it is shown that exchange property of the bases in a vector space does not hold for minimal resolving sets of wheels with $k$-consecutive missing spokes denoted by $W(n,k)$.

FLAG AND ANTI-FLAG TRANSITIVE DESIGNS

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(Joint work with John Bamberg, Alice Devillers, Cheryl Praeger)

Motivated by the study of 4-distance transitive graphs, Devillers and Praeger recently classified a class of highly symmetric designs known as pairwise transitive designs. In this talk I will discuss a generalisation of pairwise transitive designs known as FAF-transitive designs; these admit a group of automorphisms acting transitively on the sets of flags and anti-flags. Emphasis will be placed on FAF-designs which arise in coding theory and finite geometry.
NEW ESTIMATES FOR EXPECTATIONS OF EXPONENTIAL FUNCTIONS AND ITS APPLICATION IN ANALYTIC COMBINATORICS

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Many enumeration problems in combinatorics, including such fundamental questions as the number of regular graphs, can be expressed as high-dimensional complex integrals. The asymptotic behaviour could be found by concentrating the integral in a small region and then approximating the integrand inside that region. Typically, the integrals that occur are of the form

$$I = \int_\Omega \exp(-x^T A x + f(x)) \, dx,$$

where $A$ is a positive-definite real matrix and $f(x)$ is a polynomial of low degree with complex coefficients.

Let $X$ be a random variable whose distribution is given by gaussian density $C \exp(-x^T A x)$ truncated to domain $\Omega$. Then we have

$$I = C^{-1} \mathbb{E} e^{f(X)},$$

so the problem is reduced to estimating $\mathbb{E} e^{f(X)}$.

In this talk we give bounds on $\mathbb{E} e^{f(X)}$ that are general and precise enough to include practically all of relevant examples previously solved in the literature and many more similar examples.

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ON THE FRACTIONAL BOXICITY OF GRAPHS

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A box in Euclidean $k$-space is the Cartesian product of $k$ closed intervals on the real line. The boxicity of a graph $G$, denoted by $\text{box}(G)$, is the minimum nonnegative integer $k$ such that $G$ can be isomorphic to the intersection graph of a family of boxes in Euclidean $k$-space. The concept of boxicity of graphs was introduced by F.S. Roberts (1969) and has an application to a problem of niche overlap in ecology for example. Adiga et al. (2013) presented a lower bound for the boxicity of a graph $G$ as follows: $\text{box}(G) \geq \frac{|E(G)|}{|E(\overline{G})|}$, where the symbol $\overline{G}$ denotes the complement of $G$ and $I_{\text{min}}$ is an interval supergraph of $G$ on $V(G)$ with the minimum number of edges among all such interval supergraphs of $G$.

In this talk, we review the above lower bound in the context of fractional graph theory and present the fractional analogue related to boxicity. Since the boxicity of a graph can be considered as the value of an integer program, we define the fractional boxicity of a graph $G$, denoted by $\text{box}_f(G)$, as the value of the linear relaxation of the integer program for boxicity. We show that the inequalities $\text{box}(G) \geq \text{box}_f(G) \geq \frac{|E(\overline{G})|}{|E(I_{\text{min}})|}$ hold.
ADJACENCY SPECTRUM OF COMPLETE JOIN OF REGULAR GRAPHS

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Let $G_i$ be a $q_i$-regular graph of order $p_i$ for $i = 1, \ldots, s$. By $G_1 \vee \cdots \vee G_s$ we denote the complete join of $G_1, \ldots, G_s$ which is a graph that

$$V(G_1 \vee \cdots \vee G_s) := \bigcup_{i=1}^s V(G_i)$$

and

$$E(G_1 \vee \cdots \vee G_s) := \bigcup_{i=1}^s E(G_i) \cup \{uv \mid u \in V(G_i), v \in V(G_j), i \neq j\}.$$ 

In the case that $q_i = 0$ for $i = 1, \ldots, s$ we are dealing with a complete $s$-partite graph. F. Esser and F. Harary in [2], proved several properties of the spectrum of a multipartite graph. More results on the spectrum of such graphs can be found in [1, 3]. In this talk, it will be presented some several properties of the spectrum of a complete join of regular graphs analogous with those of a multipartite graphs.

Bibliography


**A NEW PROOF OF THE TIGHT CUT LEMMA**

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In deriving their characterization of the perfect matchings polytope, Edmonds, Lovász, and Pulleyblank introduces the so-called *Tight Cut Lemma* as the most challenging part of their work.

**Tight Cut Lemma.** Any tight cut in a brick is trivial.

A graph is a **brick** if deleting any two vertices results in a connected graph with a perfect matching. A cut is **tight** if it shares exactly one edge with any perfect matching. A tight cut is **trivial** if it is a star cut.

The Tight Cut Lemma in fact characterizes the **bricks** as the fundamental building blocks of that constitute a graph in the polyhedral study of matchings via the inductive operation the **tight cut decomposition**. Edmonds et al. revealed the significance of **bricks** in studying the matching polytope and lattice, and since then bricks have been always the central notion. Their proof was given by linear programming argument, whereas the statement itself consists of purely graph theoretic notions only. Szigeti later gave a purely graph theoretic proof using the theory of optimal ear-decomposition proposed by Frank.

In this talk, we introduce a new purely graph theoretic proof using, as the only preliminary result, the canonical decomposition recently developed by Kita. As this canonical decomposition is established from scratch via the most elementary discussion on matchings (1-matchings), our new proof is also distinguished as **purely matching theory closed**. As the Tight Cut Lemma and the main applications are purely 1-matching theoretic, we believe that our proof has a quite reasonable nature. We also believe that our new proof of the Tight Cut Lemma provides a highly versatile example of how to handle bricks.

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**CRITICAL EXPONENTS OF DOWLING MATROIDS**

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(Joint work with Keisuke Shiromoto)

The critical exponent of a matroid is one of the important parameters in matroid theory and is strongly related to the Critical Problem. In 1996, Joseph Kung gave a necessary and sufficient condition that an weight-1 Dowling geometry \( B_{n,t}(q) \) over a finite field \( \mathbb{F}_q \) has critical exponent \( n - 1 \), and he gave a sufficient condition that \( B_{n,t}(q) \) has critical exponent \( n - 2 \) by using a linear algebraic approach. In this talk, we give a necessary and sufficient condition that \( B_{n,t}(q) \) has critical exponent \( n - 2 \) by using a coding theoretical approach.
THE CHROMATIC NUMBER OF THE SQUARE OF THE 8-CUBE

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(Joint work with Patric R. J. Östergård)

A cube-like graph is a Cayley graph for $\mathbb{Z}_2^n$. In particular, the $k$th power of the $n$-cube, $Q_n^k$, is the graph over $\mathbb{Z}_2^n$ that has edges between two vertices if their Hamming distance is at most $k$. An independent set in $Q_n^k$ corresponds to a binary code of length $n$ and minimum distance $k + 1$. The chromatic number $\chi(Q_n^k)$ is thus the number of such codes needed to partition the $n$-dimensional Hamming space.

The maximum size of a one-error-correcting binary code of length 8 is 20, which gives a lower bound $\chi(Q_8^2) \geq \lceil 2^8 / 20 \rceil = 13$. On the other hand, 14-colorings were found independently by Hougardy in 1991 and Royle in 1993. We discuss a computer search that was used to find a 13-coloring of $Q_8^2$, thus proving that $\chi(Q_8^2) = 13$.

THE PATH LENGTH OF DIGITAL SEARCH TREES UNDER THE MARKOV MODEL

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(Joint work with Ralph Neininger and Wojciech Szpankowski)

Digital trees, such as tries, PATRICIA tries and digital search trees are tree-like data structures on words. Typically, a theoretical analysis of these kinds of data structures is done under a random input assumption. In this talk we assume that the input consists of $n$ independent and identically distributed words, each word being the sequence of states visited by a Markov chain on $\{0, 1\}$. A digital search tree storing these words then is a random rooted tree with $n$ vertices and a recursive growth rule determined by the input model.

The talk is focused on the analysis of the (internal) path length of such a random tree as the number of vertices (i.e. stored words) tends to infinity. This quantity is related to the average search cost and the construction cost of a digital search tree as well as the analysis of the Lempel-Ziv’78 lossless data compression algorithm.
1-ROTATIONAL SOLUTIONS TO THE TWOFOLD OBERWOLFACH PROBLEM

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For each integer $n \geq 3$ and each 2-regular graph $F$ of order $n$, the Oberwolfach problem, $OP(F)$, asks for a 2-factorisation of the complete graph of order $n$ in which each 2-factor is isomorphic to $F$. Originally posed in the 1960’s, the Oberwolfach problem is still only decided for a very small fraction of possible parameters. The twofold Oberwolfach problem, $OP_2(F)$, is a variant of the problem which asks for a 2-factorisation of the complete graph of order $n$ in which each edge has multiplicity 2. One possible approach to the problem is to look for a highly symmetric solution called a 1-rotational solution, which is generated by a 2-factor called a starter. We show that a solution to $OP_2(F)$ always exists when $F$ consists of one 3-cycle and $t$ 2-cycles, with the solution given by a starter when $t \not\equiv 2 \mod 4$. We also present the results of a computational search for starters for all undecided instances of $OP_2(F)$ for orders $10 \leq n \leq 32$.

COPS, ROBBERS, AND INFINITE GRAPHS

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Pursuit-evasion based searching, also known as the game of cops and robbers is a game on a graph between two players, $C$ (the cop) and $R$ (the robber). The rules are as follows: In the first round both $C$ and $R$ choose a starting vertex, in each consecutive round they are allowed to move to a neighboring vertex. The cop wins the game, if after some finite number of steps $C$ and $R$ occupy the same vertex, otherwise the robber wins.

A basic question related to this game is to characterize the class of cop win graphs, that is, graphs for which $C$ has a winning strategy. Nowakowski and Winkler, and independently Quilliot showed that in the case of finite graphs these are exactly the graphs which can be constructed according to a certain recursive procedure.

For infinite graphs Chastand et al introduced a modified winning criterion for which they believed the same to be true. We disprove their conjecture and explore further modifications which may lead to an extension of the above result from finite to infinite graphs.
CLASSIFYING BENT FUNCTIONS BY THEIR CAYLEY GRAPHS

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It is well known [1] that if a bent function $f : \mathbb{Z}_2^m \rightarrow \mathbb{Z}_2$ has $f(0) = 0$, then it has a strongly regular Cayley graph whose parameters $(v_m, k_m, \lambda_m, \lambda_m)$ depend only on $m$:

$$(v_m, k_m, \lambda_m) = (4^m, 2^{2m-1} \pm 2^{m-1}, 2^{2m-2} \pm 2^{m-1}).$$

It is perhaps less well known that even if two such Cayley graphs have the same strongly regular graph parameters, they are not necessarily isomorphic. This talk examines the concepts of Cayley equivalence and extended Cayley equivalence of bent functions, and compares these equivalence relations to the better known concepts of affine equivalence and extended affine equivalence. The relationship between two-weight codes, bent functions and strongly regular graphs is also touched on.

Bibliography


WHAT IS RAMSEY-EQUIVALENT TO THE CLIQUE?

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(Joint work with Thomas Bloom)

The concept of Ramsey-equivalence was introduced only recently, in 2010, by Szabó, Zumstein and Zürcher. A surprising fact, proved by Fox, Grinshpun, Person, Szabó and myself, is that the only connected graph that is Ramsey-equivalent to the complete graph $K_k$ on $k$ vertices is $K_k$ itself. That is, removing a single edge from $K_k$, or adding a single pendant edge to it, completely changes the Ramsey behaviour. On the other hand, the graph $K_k + s \cdot K_t$, consisting of a copy of $K_k$ and $s$ vertex disjoint copies of $K_t$, is Ramsey-equivalent to $K_k$ for all $2 \leq t \leq k - 2$ and suitable $s = s(k, t)$. However, it remained open to decide whether $K_k$ is Ramsey-equivalent to $K_k + K_{k-1}$ for $k \geq 4$. We prove that this is indeed the case.

In this talk, I will define all necessary concepts, survey previous work on Ramsey-equivalence, and present exciting open questions in the area.
EXISTENCE AND CONSTRUCTIONS OF CYCLIC GRID-BLOCK DESIGNS

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(Joint work with Junya Satoh and Masakazu Jimbo)

The notion of grid-block designs originated from the experimental designs for DNA library screening as follows: For a $v$-set $V$, let $B$ be a collection of $r \times k$ arrays with $rk$ different entries in $V$. A pair $(V, B)$ is called an $r \times k$ grid-block design if every pair of distinct points in $V$ occurs exactly once in the same row or in the same column of a grid-block of $B$. Moreover, $(V, B)$ is cyclic, if $B$ admits a cyclic group of order $v$ as its automorphism.

In this talk, by utilizing cyclotomic methods, we investigate the existence and constructions of cyclic $2 \times k$ grid-block designs for odd $k$. Moreover, from an algebraic number theoretic viewpoint, we will show that a cyclic $(p, 2 \times k, 1)$ grid-block designs holding the “cyclotomic structure” exits for an infinitely many number of prime $p$.

THE ERDŐS-KO-RADO THEOREM, GENERALISATIONS AND BEYOND

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Extremal Set Theory is a branch of Extremal Combinatorics where one characterises the maximum size of a family of sets with certain restriction on them. The Erdős-Ko-Rado Theorem is a classical result in Extremal Set Theory and since its discovery, it has been extensively researched and generalise.

In this talk an introduction of the Erdős-Ko-Rado Theorem is given as well as some generalisations and analogous results for other structures such as vector spaces over a finite field. Open problems as well as new possible directions of research are given for a particular generalisation of the Erdős-Ko-Rado Theorem.
THE ENUMERATION OF CYCLIC MNOLS

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(Joint work with F. Demirkale and D. Donovan)

In this talk we will describe collections of mutually nearly orthogonal Latin squares (MNOLS), which come from a modification of the orthogonal condition on the famous mutually orthogonal Latin squares. In particular, we present the maximum $\mu$ such that there exists a collection of $\mu$ cyclic MNOLS of order $n$ for $n \leq 16$, as well as providing a full enumeration of collections of $\mu$ cyclic MNOLS of order $n$ under a variety of equivalences with $n \leq 16$. This will resolve in the negative a conjecture of Li and van Rees that proposed the maximum $\mu$ for which a set of $\mu$ cyclic MNOLS of order $n$ exists is $\lceil n/4 \rceil + 1$.

ALGORITHMS AND COMPLEXITY FOR TURAEV-VIRO INVARIANTS

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(Joint work with Benjamin A. Burton and Jonathan Spreer)

The Turaev-Viro invariants are a powerful family of topological invariants for distinguishing between different 3-manifolds. They are invaluable for mathematical software, but current algorithms to compute them require exponential time.

The invariants are parameterised by an integer $r \geq 3$ and are defined as sums of exponential size over “states” on the triangulation. We present in this talk an explicit fixed-parameter tractable algorithm for computing Turaev-Viro invariants for arbitrary fixed $r$. The algorithm is parameterised by the treewidth of the dual graph of the triangulation, i.e. the graph representing tetrahedra of the triangulation and their adjacency. We finally show through concrete implementation and experimentation that this algorithm is practical—and indeed preferable—to the prior state of the art for real computation.
**SOME RESULTS ON PLANAR HYPOHAMILTONIAN GRAPHS**

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(Joint work with Mohammadreza Jooyandeh, Patric Östergård, Ville Pettersson and Carol Zamfirescu)

A graph is hypohamiltonian if it is not hamiltonian yet every graph obtained by deleting one vertex is hamiltonian. We discuss some recent results on planar hypohamiltonian graphs, including the first examples with girth 5.

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**THE SWITCHING-STABLE GRAPHS**

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(Joint work with Brendan McKay)

The classical reconstruction problem asks when a graph $G$ can be reconstructed from its *deck*, where the deck consists of *cards* showing each of the vertex-deleted subgraphs of $G$. Stanley, Bondy and others introduced variants of this problem, where the cards instead show the graphs obtained by *switching* $G$. While investigating switching reconstruction problems, Bondy asked which graphs have the property that every possible switching produces a graph that is equivalent to the original graph. Such graphs are said to be *switching-stable*.

We answer Bondy’s question for two types of switching. First, we let the edges of $G$ be coloured with two colours and define the switching operation to be the interchange of the colours of the edges incident to a specified vertex and classify all switching-stable 2-edge-coloured graphs. Then, we let $G$ be a digraph and define the switching operation to be the reversal of the directions of the edges incident to a specified vertex and classify all switching-stable digraphs. For both variants of switching, we consider the problem for two different types of equivalence.
CERTIFICATES OF TUTTE EQUIVALENCE

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(Joint work with Graham Farr and Kerri Morgan)

A graph polynomial is a polynomial which gives information about some properties of a graph. A certificate, for a particular graph polynomial, is a sequence of graph operations based on properties of the graph polynomial. The Tutte polynomial is the central member of a large family of graph polynomials. Two graphs are Tutte equivalent if they have the same Tutte polynomial. Certificates can explain Tutte equivalences without computing Tutte polynomials. Short certificates give proofs of Tutte equivalence which can be verified efficiently. We developed a certificate searching program and investigated Tutte equivalence for simple graphs of order \( \leq 8 \) and found all certificates of length \( \leq 4 \). We also studied some interesting sets of Tutte equivalent graphs and found short certificates for these cases. In this talk we will present our results and discuss the relationship between certain graphs that are Tutte equivalent.

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A NON-CLASSIFICATION THEOREM FOR 2-ARC-TRANSITIVE GRAPHS

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(Joint work with Eric Swartz, Gabriel Verret)

It would be great to have a classification of 2-arc-transitive graphs! Sadly, this is probably out of reach. However, there are many results on classifications of 2-arc-transitive graphs of certain orders, valencies, girth, etc. In this talk, I’ll describe a recent result of Eric Swartz, Gabriel Verret and myself where we do something slightly different. Rather than classify all the possible graphs, we show for fixed \( n \) and \( k \), the family of 2-arc-transitive graphs of order \( kp^n \) is finite, with \( p \) a prime, once the valency is big enough (relative to \( n \)). So, our “non-classification” shows that the most interesting 2-arc-transitive graphs are those of small valency (relative to \( n \)).
Some results related to the critical problem for matroids

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In 1941, Brooks characterized the graphs for which the chromatic number exceeds the maximum degree. In 1965, Erdős noted that every loopless graph has a bipartite subgraph that uses at least half the edges. In 1992, Maffray determined the graphs with no odd cycles of length exceeding three. The critical problem of Crapo and Rota provides a unified framework for viewing all three of these results. Using this framework, this talk will present matroid generalizations of these three graph results.

Representing 3-manifolds as decompositions of graphs

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(Joint work with Benjamin A. Burton)

Manifolds are a topological abstraction of flat surfaces, and are often represented as triangulations. They are of interest in many fields, including biology, astronomy and computational modelling. Triangulations are usually described as a set of simplices, along with the identifications, or “gluings”, used to pair up the faces.

In this talk, I will present a representation of triangulations as decompositions of graphs. This work follows on from that presented at 36ACCMCC, and I will explain the new representation as well as how it helps with isomorphism identification, census enumeration and also potential links to other fields like high-energy physics.
List Backbone Coloring of Paths and Cycles

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Let $H$ be a subgraph of $G$ in a graph pair $(G, H)$. A backbone $k$-coloring for $(G, H)$ is a proper coloring of $G$ by the set of colors $\{1, 2, \ldots, k\}$, adding a condition that colors assigned to adjacent vertices in $H$ must differ by at least two. A list assignment $L$ is a mapping that assigns a set of positive integers $L(v)$ to each vertex $v$ in $G$. A $k$-assignment $L$ of $G$ is a list assignment $L$ with $|L(v)| = k$ for each vertex $v$. If there is a backbone coloring $c$ of $G$ such that $c(v) \in L(v)$, then $(G, H)$ is backbone $L$-colorable. The backbone choice number of $(G, H)$, denoted by $ch_{BB}(G, H)$, is the smallest integer $k$ such that $G$ is backbone $L$-colorable for each $k$-assignment $L$.

For a path or a cycle $G$, results of Bu et al. [1] implied that the possible maximum value of $ch_{BB}(G, H)$ is $9$. In this research, we show that the possible maximum value of $ch_{BB}(G, H)$ can be reduced to $5$. Moreover, we investigate exact values of $ch_{BB}(G, H)$ where $G$ is a path or a cycle in all possible structures of subgraphs $H$ of $G$.

Bibliography


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3-Flows with Large Support

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(Joint work with M. DeVos, J. McDonald, E. Rollova, and R. Samal)

One of the fundamental open problems in graph theory is Tutte’s 3-Flow Conjecture, which states that every 4-edge-connected graph admits a nowhere-zero 3-flow. We relax the notion of “nowhere-zero” and instead consider flows which have as few zeros as possible. We prove that every 3-edge-connected graph has a 3-flow which has at most $\frac{1}{5}$ of the edges with flow value zero. This bound is best possible, in that it is achieved by $K_4$. 

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ARC-TYPES OF CAYLEY GRAPHS

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This is a sequel to the talk given by Marston Conder. The arc-type of a vertex transitive graph is a partition of the graph’s valency as a sum of the lengths of the orbits of a vertex-stabilizer on the neighbourhood of that vertex. We use parentheses in the partition to denote paired orbits. It has been shown by Conder, Pisanski and Žitnik that every arc-type except for $2 = 1+1$ and $2 = (1+1)$ is realized by some vertex transitive graph [1]. We advance this result by showing that every arc-type except for $2 = 1+1$ and $2 = (1+1)$ is realized by some Cayley graph.

Bibliography


DETERMINISTIC CONSTRUCTION OF APPLIED TANNER GRAPHS WITH NO SHORT CYCLES

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Low-density Parity-check (LDPC) codes are a high-performing class of error correcting codes which are used in applications such as the transmission of digital television and Ethernet connections. The underlying structure of an LDPC code is a matrix which is the adjacency matrix of a bipartite graph, called a Tanner graph. In order to ensure error correcting efficiency, these matrices must satisfy a number of conditions which correlate with properties of the Tanner graph. A few important conditions are that the graph must contain no short cycles and must have no small degree vertices. The number and complexity of these conditions has led to limited success in regard to deterministic construction methods in the past. Hammons, Kumar, Calderbank, Sloane and Solé (1994) demonstrate that the construction process of some error correcting codes can be simplified by transforming the matrices from a binary structure to one over $\mathbb{Z}_4$ using the Gray map. We show that it is possible to deterministically construct efficient LDPCs through the use of sequences over $\mathbb{Z}_4$, such as De Bruijn or Skolem sequences.
IDENTIFYING FAMILIES OF MATROIDAL CYCLIC FLATS

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A matroid is a kind of finite, discrete geometric structure, made up of points arranged in an arbitrary-dimensional space. Study of matroids has revealed a number of distinct, equivalent axiom formulations that describe them. One such axiom system defines matroids in terms of subsets called cyclic flats, which form a lattice under subset inclusion, and an associated integer-valued rank function on the cyclic flats.

This raises the question: given a lattice of subsets of a finite set, when is it possible to define a rank function on them that will comply with the cyclic flat axiom system, resulting in a matroid? This can be solved computationally, but it is an integer programming problem, which suggests that there may be no polynomial-time solution. However, we believe that the matrix of constraints has sufficient structure that its vertices will always be integral, meaning that the problem becomes one of linear programming.

DOES DEFINABILITY EQUAL RECOGNIZABILITY FOR HYPERGRAPHS?

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Hlineny has shown that if a class of representable matroids of bounded branchwidth can be defined in monadic second order logic it can be recognized by a tree automaton. In an attempt to obtain a partial converse to Hlineny’s Theorem we have been studying binary matroids of bounded pathwidth. As part of this process we are investigating hypergraphs of bounded pathwidth and asking whether if such structures are recognizable they must be logically definable. This question has been answered positively for graphs by Kabanets.
ON THE DISTANCES BETWEEN LATIN SQUARES

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(Joint work with Nick Cavenagh)

A Latin square is an \( n \times n \) array of symbols \( \{0, 1, \ldots, n-1\} \) in which each symbol occurs precisely once in each row and once in each column. In this talk, we show that for each Latin square \( L \) of order \( n \), there exists a Latin square \( L' \neq L \) of order \( n \) such that \( L \) and \( L' \) differ in at most \( 8\sqrt{n} \) cells. Note that a trivial upper bound is \( 2n \), obtained simply by swapping any pair of rows. While it is conjectured to be of the order \( O(\log n) \), our result is the first upper bound which is \( o(n) \).

We also improve previous bounds on the size of the smallest defining set in a Latin square and show that it is \( \Omega(n^{3/2}) \), where a defining set is a partial Latin square which has a unique completion to a Latin Square.

PROFILE OF REPETITIONS IN RANDOM SEQUENCES

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(Joint work with Philippe Chassignet)

Many repetitive structures can be found in genomes. Therefore, it is a key issue in several genomic problems to distinguish biologically significant events from random repetitions in sequences. For instance, knowledge about the length of a maximal repeat is crucial in re-sequencing and assembly, notably the design of algorithms that rely upon de Bruijn graphs.

Length of repetitions and their distribution should depend on typical parameters such as the background probability model, the size of the alphabet, the length of the sequence, ... A work by Park and colleagues in 2009 opened the way for binary alphabets. Our combinatorial approach avoids the Poissonization-dePoissonization cycle that was used. It extends to non-binary alphabets. In that case, there is no closed formula for the asymptotic behaviour. Nevertheless, the Lagrange multipliers allow to derive it as the solution of an equation that can be computed numerically.

Explicit and computable bounds for the profile are provided. Three domains can be observed for the behaviour. A good convergence is shown in practice for random texts and differences between the model and the observation are studied on the special case of ARCHAEO genomes.
FORBIDDING RAINBOW-COLORED STARS

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(Joint work with Carlos Hoppen, Hanno Lefmann and Knut Odermann)

We consider an extremal problem motivated by a question of Erdős and Rothschild and by a paper of Balogh, who considered edge-colorings of graphs avoiding fixed subgraphs with a prescribed coloring. Given \( r \geq t \geq 2 \), we look for \( n \)-vertex graphs that admit the maximum number of \( r \)-edge-colorings such that at most \( t - 1 \) colors appear in edges incident with each vertex. For large \( n \), we show that, with the exception of the case \( t = 2 \), the complete graph \( K_n \) is always the unique extremal graph. Since the techniques used to prove this result may be adapted to other patterns, we state our results in greater generality. In particular, to derive this generalization, we show that in any graph with ‘many’ edges, there is an ‘almost spanning’ subgraph with a ‘large’ number of subgraphs of any bounded degree sequence satisfying a density constraint, which seems to be of independent interest.

ERDŐS-RÉNYI THEORY FOR ASYMMETRIC DIGRAPHS

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(Joint work with Masanori Sawa and Masakazu Jimbo)

In 1963, Erdős and Rényi defined a measure of asymmetry of undirected graph with \( n \) vertices. They showed an upper bound and that this bound is asymptotically best possible. Moreover, they showed that countable random graphs are almost surely isomorphic to a symmetric graph, namely Rado graph \( R \). After that many paper investigated the structure of \( R \).

In this talk, we give a digraph analogue of these results and we deal with digraphs with no loops, multi-edges and parallel-edges. First, we define the asymmetry number of digraphs with \( n \) vertices and we show an upper bound. Then we introduce an our result which implies that this bound is asymptotically best possible used probabilistic methods. Next, we consider countable digraphs and show that countable random digraphs are almost surely isomorphic to a symmetric digraph called random oriented graph \( RO \). Moreover we prove \(|\text{Aut}(RO)| = 2^{\aleph_0}\) by some methods and introduce some our graph-theoretic results of \( RO \).
Cyclotomic numbers and equations over finite fields

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(Joint work with Tai Do Duc and Ka Hin Leung)

Let \( q = p^a = ef + 1 \) where \( p \) is a prime and let \( C_0 \) be the subgroup of order \( f \) of the multiplicative group of the finite field \( \mathbb{F}_q \). Let \( g \) be a primitive element of \( \mathbb{F}_q \) and write \( C_i = C_0 g^i \), \( i = 1, \ldots, e - 1 \). The number of pairs \((x, y)\) with

\[
1 + x = y, \ x \in C_i, y \in C_j
\]
is denoted by \((i, j)\). The \((i, j)\)'s are called cyclotomic numbers of order \( e \).

If \( f \) is small compared to \( p \), one expects the \((i, j)\)'s to be small, too. For instance, it was shown in [1] that \((0, 0) \in \{0, 2\}\) if \( p \) is large enough compared to \( f \).

We prove results of this form for all cyclotomic numbers (not restricted to \((0, 0)\)) and provide effective bounds on \( p \). For example, we show that

(i) \((0, 0) = 0\) if \( p > 3f/2 \) and \( f \neq 0 \) (mod 6),

(ii) \((i, j) \in \{0, 1, 2\}\) whenever \( i \neq j \) and \( p > 2^{2f-2} \).

These results have consequences for the solvability of some equations over finite fields. For instance, if the assumptions of (i) hold, then \( x^e + y^e = z^e \), \( x, y, z \in \mathbb{F}_q \), is only possible if \( xyz = 0 \).


Cretan matrices

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An application in image processing (compression, masking) led to the search for orthogonal matrices whose elements have modulus \( \leq 1 \) which have maximal or high determinant.

A Cretan\((n)\) matrix, of order \( n \), is an orthogonal matrix whose elements have moduli \( \leq 1 \). This paper gives new constructions for Cretan\((n)\) matrices constructed using regular Hadamard matrices, SBIBD\((4t + 1, k, \lambda)\), weighing matrices, generalized Hadamard matrices and the Kronecker product. We discuss the importance of the congruence class of \( n \) modulo 4 and give an inequality for the radius. We show Cretan matrices exist for every order \( n \geq 3 \), however it is challenging to find non-trivial examples in many cases.
ON 3-CONNECTED ES-SPLITTING BINARY MATROIDS

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Let $M$ be a binary matroid on a set $E$ and let $X$ be a subset of $E$ with $e \in X$. Suppose $A$ is a matrix representation of $M$ over GF(2). Let $A_X^e$ be a matrix obtained from $A$ by adjoining an extra row to $A$ with entries zero everywhere except in the columns corresponding to the elements of $X$ where it takes the value 1 and then adjoining two columns labelled $a$ and $\gamma$ to the resulting matrix such that the column labelled $a$ is zero everywhere except in the last row where it takes the value 1, and $\gamma$ is the sum of the two column vectors corresponding to the elements $a$ and $e$. The vector matroid of the matrix $A_X^e$ is denoted by $M_X^e$. The transition from $M$ to $M_X^e$ is called an es-splitting operation. We call the matroid $M_X^e$ as es-splitting matroid.

The es-splitting operation on a 3-connected binary matroid, in general, may not preserve the 3-connectedness of the matroid. In this paper, we provide sufficient conditions for a 3-connected binary matroid to yield a 3-connected binary matroid under the es-splitting operation when $|X| = 2$. We also provide some applications of the es-splitting operation.

LOCAL METRIC DIMENSION OF SUBGRAPH-AMALGAMATION OF GRAPHS

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(Joint work with G. A. Barragán Ramírez, Suhadi W. Saputro, and Saladin Uttunggadewa)

A vertex $v \in V(G)$ is said to distinguish two vertices $x, y \in V(G)$ of a nontrivial connected graph $G$ if the distance from $v$ to $x$ is different from the distance from $v$ to $y$. A set $S \subseteq V(G)$ is a local metric set for $G$ if every two adjacent vertices of $G$ are distinguished by some vertex of $S$. A local metric set with the minimum cardinality is called a local metric basis for $G$ and its cardinality, the local metric dimension of $G$, denoted $\dim_l(G)$.

Let $\{G_i\}_{i=1}^n$ be a family of graphs with a common induced subgraph $J$ (with $J_i$ as the copy of $J$ in $G_i$), where $V(G_i) = \{u_i^l\}, V(J) = \{v_i\}, V(J_i) = \{v_i^l\}$. We define the subgraph-amalgamation of $\{G_i\}$ over $J$, denoted by $\Pi(\{G_i\|J\})$, as the graph with vertex set $V(\Pi(\{G_i\|J\})) = \bigcup_{i=1}^n V(G_i - J_i) \cup V(J)$ and edge-set $E(\Pi(\{G_i\|J\})) = \bigcup_{i=1}^n E(G_i - J_i) \cup E(J) \cup \bigcup_{i=1}^n \{u_i^l v_i^l|u_i^l \in V(G_i), v_i^l \in V(J_i)\}$.

In this paper we study several tight bounds for local metric dimension of subgraph-amalgamation of graphs.
**OPTIMAL TWO-LEVEL CHOICE DESIGNS FOR ANY NUMBER OF CHOICE SETS**

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Discrete choice experiments are widely used in various areas including marketing, transport, environmental resource economics and public welfare analysis. For two-level choice experiments, we derive a simple form of the information matrix of a choice design for estimating the main effects and find $D$-optimal choice designs for any number of choice sets. We also obtain $MS$-optimality bound and corresponding designs for this setup. It is shown that the optimal designs under the main effects model are also optimal under the broader main effects model. We find that optimal choice designs with a choice set size two often outperform their counterparts with larger choice set sizes.

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**CHARACTERIZATION OF UNIQUELY (2, $t$)-LIST COLORABLE FOR GRAPHS WITH A SUBGRAPH $\theta_{p,q,r}$**

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A $(2, t)$-list assignment $L$ of a graph $G$ assigns a list of 2 colors available at each vertex $v$ in $G$ and $|\bigcup_{v \in V(G)} L(v)| = t$. A graph $G$ is said to be uniquely $(2, t)$-list colorable if it admits a $(2, t)$-list assignment from which $G$ has a unique list coloring. Let $\theta_{p,q,r}$ denote a graph obtained by identifying all beginnings and identifying all ends of 3 disjoint paths having $p$, $q$ and $r$ edges respectively. The result of Mahdian et al. [2] provided that, if $G$ is uniquely 2-list colorable, then $G$ has a subgraph $\theta_{p,q,r} \neq \theta_{2,2,2}$. Ganjari et al. [1] showed that $\theta_{p,q,r} \neq \theta_{2,2,2}$ is uniquely $(2,3)$-list colorable and used this result to prove a graph $G$ is uniquely 2-list colorable if and only if it is uniquely $(2, t)$-list colorable where $t = \max\{3, \chi(G)\}$. Additional, this result is a lower bound of a number $t$.

In this paper, we show that a graph $G$ with a subgraph $\theta_{p,q,1}$ is uniquely $(2, t)$-list colorable if and only if $\max\{3, \chi(G)\} \leq t \leq |V(G)| - 1$.

**Bibliography**


HAMiLTON DECOMPOSITIONS OF LINE GRAPHS

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(Joint work with Darryn Bryant and Barbara Maenhaut)

We show that the line graph of a Hamilton decomposable graph is Hamilton decomposable, thus settling a conjecture of Bermond (Problem 97, Discrete Math. 71, 1988).

ON EDGE-TRANSITIVE TETRAVALENT METACIRCULANTS

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A graph $\Gamma$ is called edge-transitive if the automorphism group $\text{Aut} \Gamma$ of $\Gamma$ is transitive on the edge set. It is half-transitive if $\text{Aut} \Gamma$ is transitive on the edge set, vertex set but not transitive on the arc set. A graph is called a weak metacirculant if its automorphism group contains a transitive metacyclic group.

In this talk, we will talk about edge-transitive tetravalent weak metacirculants. It is shown that an edge-transitive tetravalent weak metacirculant is a normal Cayley graph unless its automorphism group is one of four special insoluble groups. If further, the graph is a Cayley graph of a non-abelian metacyclic group, then it is half-transitive.
Séparation index of graphs and stacked 2-spheres

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(Joint work with Benjamin Burton, Basudeb Datta, and Nitin Singh)

In 1987, Kalai proved that stacked spheres of dimension $d \geq 3$ are characterised by the fact that they attain equality in Barnette’s celebrated Lower Bound Theorem. This result does not extend to dimension $d = 2$. In this talk, I will present a characterisation of stacked 2-spheres using what is called the separation index. Namely, the separation index of a 2-sphere triangulation is maximal if and only if it is stacked. In addition, I will show that, amongst all $n$-vertex 2-sphere triangulations, the separation index is minimised by some flag sphere.

This characterisation of stacked 2-spheres is then applied to settle the outstanding 3-dimensional case of the Lutz-Sulanke-Swartz conjecture that “tight-neighborly triangulated manifolds are tight”. For dimension $d \geq 4$, the conjecture is already known due to Novik-Swartz and Effenberger.

Collineations of finite 2-affine planes

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A 2-affine (or bi-affine) plane is a non-trivial linear space such that for each line $L$ and each point $p \notin L$ there are 1 or 2 lines through $p$ that do not meet $L$. A finite 2-affine plane has order $n$ if $n$ is the maximum number of points on a line. M. Oehler [Endliche bi-affine Inzidenzebenen, Geom. Dedicata 4 (1975) 419–436] determined all finite 2-affine planes. They fall into four classes: affine planes, affine planes from which a point has been removed, affine planes from which a line and all its points have been removed, or one of four sporadic planes of order 2, 3 or 4.

Finite 2-affine planes of order $n$ of the first three kinds are embedded into projective planes of the same order $n$. We consider these projective extensions and show that they are uniquely determined by the 2-affine plane (whereas, in the third case, the affine extensions not necessarily are). Furthermore, each collineation of the 2-affine plane is induced by a collineation of its projective extension. We also consider the collineation groups of the four sporadic 2-affine planes.
**Power Domination in De Bruijn and Kautz Digraphs**

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(Joint work with Thomas Kalinowski, Paul Manuel, Cyriac Grigorious, Mirka Miller, Bharati Rajan)

A vertex set $W$ is said to be critical if there is no vertex outside $W$ which has fewer than two neighbors in $W$. If $W$ is critical, but no proper subset of $W$ is critical, then we call $W$ minimal critical. A vertex set $S$ is a propagating set if and only if $S \cap X \neq \emptyset$ for every minimal critical set $X$ in $G$ and the minimum cardinality of such a set is called the propagation number, denoted by $Z(G)$. A vertex set $S$ is a power dominating set if and only if $N(S) \cap W \neq \emptyset$ for every minimal critical set $W$. In this paper we solve the propagation problem and power domination problem in de Bruijn and Kautz digraphs.

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**Some Properties of Antiadjacency Matrix of Directed Graph**

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(Joint work with Wildan and B. D. Handari)

Let $G$ be a directed graph with $V(G) = \{v_1, v_2, ..., v_n\}$. An antiadjacency matrix of graph $G$ is a matrix $B = I - A$ where $A$ is a matrix adjacency of graph $G$. Thus, if $B = (b_{ij})$ then $b_{ij} = 0$ if there is an arc from $v_i$ to $v_j$, and $b_{ij} = 1$ for other cases. In this talk, we will discuss some properties of polynomial characteristic and eigen values of antiadjacency of directed graphs.
**Balanced Independent Sets in** \( PG(n, q) \)

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An independent set of a bipartite graph is called *balanced* if it contains exactly half its elements in each partite set. The problem of finding large balanced independent sets has applications in interconnection networks and coding theory, and has been studied under various guises such as the clique problem or the isoperimetric problem. We will demonstrate known bounds on the size of a maximum balanced independent set. In particular, we will consider the family of point-hyperplane incidence graphs of \( PG(n, q) \) and construct upper and lower bounds of the same order for this family. We will conclude by determining the vertex-isoperimetric number for this family of graphs.

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**Min-Max Hypergraph Partitioning and Its Application on Rapid Transit Networks**

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Due to different causes, disruptions on transportation networks often happen. For example, in railway networks, infrastructure failures, rolling stock failures, human factors of the crew, suicides and vandalism (stones or coins on the tracks), disturb railway traffic and require a quick response of the operating company.

In order to capture all the characteristics of a railway network which is structured by lines, in the railway representation by hypergraph, stations are nodes and lines are hyperedges. The combination of both concepts is called hyperstructure and captures both the binary relationship between adjacent stations and the multiple relationship of the stations belonging to the same line.

In this work, we tackle the problem of locating emergency units in a rapid transit network. For this purpose, we use a min-max hypergraph partition of the network with balanced constraints which allows us to apply in each part a center location problem to determine the stations where to establish the emergency units. The min-max hypergraph partitioning problem means partitioning a hypergraph to parts so that the maximum number of hyperedges covering each part is minimized. Balanced constraints are imposed in order to equilibrate the number of stations in each part. Exact algorithms with branch and bound technique is developed for solving this problem, and so is Fiduccia-Mattheyses Heuristic method. The work finishes with some computational experience and the application to actual metro networks.
ON TRANSVERSALS IN MULTIDIMENSIONAL ARRAYS

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Let $A$ be a $d$-dimensional array of order $n$ with integer entries. A transversal of an array $A$ is a selection of $n$ cells no two of which agree in any coordinate or share the same symbol. A $d$-dimensional latin hypercube of order $n$ is an array filled with $n$ symbols in such a way that no symbol is repeated in any 1-dimensional plane.

In [2] Wanless proposed that every latin hypercube of odd dimension or of odd order has a transversal. Generalizing the result of Balasubramanian [1], we show that every latin hypercube of even order has an even number of transversals. Also, using a modified version of the Local Lovász Lemma, we prove that if no integer appears too often in a multidimensional array $A$, then $A$ has a latin transversal.


ALGEBRAS OF INCIDENCE STRUCTURES

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An incidence structure is a standard geometric object consisting of a set of points, a set of lines and an incidence relation specifying which points lie on which lines. This concept generalises, for example, graphs, hypergraphs and projective planes. Reyes and Zolfaghari proved that the lattice of subgraphs of a graph naturally forms a double-Heyting algebra. We generalise this result and show that the lattice of point-preserving substructures of an incidence structure naturally forms an algebraic structure known as a regular double $p$-algebra.

The result given in this talk is a characterisation of the regular double $p$-algebras which are isomorphic to a lattice of point-preserving substructures. In addition to the corollary that every finite regular double $p$-algebra is isomorphic to such a lattice, a special case of the result is a standard theorem for boolean algebras: a boolean algebra is isomorphic to a powerset lattice if and only if it is complete and atomic.
ON CERTAIN SUMS INVOLVING THE LEGENDRE SYMBOL

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(Joint work with Borislav Karaivanov, Sigma Space Inc., Lanham, MD, USA)

The Legendre symbol, introduced in 1798 in conjunction with the Law of Quadratic Reciprocity
is defined as follows:
For an odd prime number \( p \) and an integer \( a \),
\[
\left( \frac{a}{p} \right) = \begin{cases} 
0, & \text{if } a \equiv 0 \pmod{p} \\
1, & \text{if } a \text{ is a quadratic residue modulo } p \text{ and } a \not\equiv 0 \pmod{p} \\
-1, & \text{if } a \text{ is a quadratic non-residue modulo } p.
\end{cases}
\]

For any prime \( p \) and integers \( a \) and \( b \), let
\[
S_p(a, b) = \sum_{l=1}^{p-1} \left( \frac{al + b}{p} \right) l.
\]

We discuss the properties of the sum with respect to all its three parameters and provide com-
putational results shedding further light on its behaviour. The work is motivated by number
of recent problems in American Mathematical Monthly dealing with particular instances of
\( S_p(a, b) \).

VERTEX-PRIMITIVE DIGRAPHS HAVING VERTICES WITH
ALMOST EQUAL NEIGHBOURHOODS

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A permutation group \( G \) on \( \Omega \) is transitive if, for every \( x, y \in \Omega \), there exists \( g \in G \) mapping \( x \)
to \( y \). The group \( G \) is called primitive if, in addition, it preserves no nontrivial partition of \( \Omega \).

Let \( \Gamma \) be a vertex-primitive digraph, that is, the automorphism group of \( \Gamma \) acts primitively on its
vertex-set. It is not hard to see that \( \Gamma \) cannot have two distinct vertices with equal neighbour-
hoods, unless \( \Gamma \) is in some sense trivial.

I will discuss some recent results about the case when \( \Gamma \) has two vertices with “almost” equal
neighbourhoods, and how these result were used to answer a question of Araújo and Cameron
about synchronising groups. (This is joint work with Pablo Spiga.)
CLASSIFYING LINEAR QUASIGROUPS

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(Joint work with Přemysl Jedlička and David Stanovský)

Finite quasigroups are the algebraic counterparts of latin squares. A quasigroup \((A,\ast)\) is said to be linear over \((A,+)\) if there are automorphisms \(\varphi, \psi\) of \((A,+)\) and \(c \in A\) such that \(x \ast y = \varphi(x) + \psi(y) + c\). Typically \((A,+)\) is more structured than \((A,\ast)\), e.g., \((A,+)\) might be an abelian group.

For certain varieties \(V\) of quasigroups it is known that all members of \(V\) are linear over appropriate algebras, and the isomorphism problem can be restated in terms of the linear representations. I will present theoretical and computational results concerning enumerations of medial, trimedial and distributive quasigroups in their linear representations, improving upon existing bounds by two orders of magnitude.

A NOTION OF PARITY FOR MOLS

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(Joint work with Nevena Francetić, Sarada Herke)

Parity is a familiar and important notion in the study of permutations. Latin squares are two dimensional permutations and also have some kind of parity. In fact, they have three basic attributes, each of which can be either even or odd. These obey a relationship which means that any two determine the third. So in information theory terms there are really just two parity bits (i.e. 4 possible parities).

What about MOLS (mutually orthogonal Latin squares)? Do they have a notion of parity? In 2012 Glynn and Byatt showed that they do. We reformulate their notion as a natural generalisation of the parity of a Latin square. We then establish an upper bound on the number of parity bits that a set of \(k\) MOLS may have. We show that this bound is achieved in infinitely many cases, but never by complete sets of MOLS. We also study the effect of natural MOLS operations on their parity. This leads to the idea of a switching class, which is a set of parities such that if any one parity in the class is achievable, then they all are.
**SUBSYSTEMS OF NETTO TRIPLE SYSTEMS**

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The Netto triple systems are a class of Steiner triple systems having order \( q = p^n \) where \( n \geq 1 \), \( p \) is prime, and \( q \equiv 7 \pmod{12} \), and there is a unique (up to isomorphism) Netto triple system for each such order. For \( q \neq 7 \), their full automorphism group acts transitively on unordered pairs of points but not on ordered pairs of points, and they are the only Steiner triple systems with this property.

Netto triple systems are block-transitive, cyclic, uniform, anti-mitre, and are block-regular if and only if \( q \equiv 7 \) or \( 31 \pmod{36} \). The elements of a field of order \( q \) form the point set of a Netto triple system of order \( q \), and the blocks can be generated from the triple \( \{0, 1, \alpha\} \) where \( \alpha \) is a primitive sixth root of unity.

In 1975 Robinson conjectured that Netto triple systems of prime order have no non-trivial subsystems. We investigate this conjecture and also study subsystems of Netto triple systems of composite order.

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**ABELIAN SQUARE-FREE GRAPH COLOURING**

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(Joint work with David R. Wood)

A word is a **square** if the first half is identical to the second half. A word is an **Abelian square** if the first half is a permutation of the second half. A graph colouring is **square-free** if no subpath has a colour sequence which is a square.

We introduce Abelian square-free graph colouring. A graph colouring is **Abelian square-free** if no subpath has a colour sequence which is an Abelian square. Both square-free and Abelian square-free paths have been studied in the context of combinatorics of words. One similarity between them is that arbitrarily long paths have square-free chromatic number 3 and Abelian square-free chromatic number 4.

A central result of square-free graph colouring is an upper bound on the square-free chromatic number by a function of maximum degree. We prove that the Abelian square-free chromatic number is not bounded by a function of maximum degree. We also prove a bound on the Abelian square-free chromatic number of trees as a function of pathwidth and show that it is tight.
LAYOUTS OF EXPANDER GRAPHS

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(Joint work with Anastasios Sidiropoulos and Vida Dujmović)

A graph is an expander if, loosely speaking, it has no separator of size $o(n)$. Expanders are of fundamental importance in diverse areas of mathematics and computer science. This talk will be a gentle (proof-free) introduction to the topic of expander graphs. The focus will be on the following question: for which graph-theoretic properties does there exist a family of expanders satisfying the given property? For example, it is well known that there are bipartite expanders with bounded degree. Bourgain and Yehudayoff went much further by recently constructing so-called $O(1)$-monotone bipartite expanders. We show how any $O(1)$-monotone bipartite expander can be manipulated to produce a 3-monotone bipartite expander, which is best possible. We then show that the same graphs admit 3-page book embeddings, 2-queue layouts, 4-track layouts, and have simple thickness 2. All these results are best possible. See arXiv:1501.05020 for the paper.

UNIFORM GENERATION OF RANDOM REGULAR GRAPHS

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(Joint work with Jane Gao)

We develop a new approach for uniform generation of random graphs with given degrees, and apply it to derive a uniform sampling algorithm for $d$-regular graphs. The algorithm can be implemented such that each graph is generated in expected time $O(nd^3)$, provided that $d = o(\sqrt{n})$. Our result significantly improves the previously best uniform sampler, which works efficiently only when $d = O(n^{1/3})$, with essentially the same running time for the same $d$. We also give a linear-time approximate sampler which, for $d = o(\sqrt{n})$, generates a random $d$-regular graph whose distribution differs from the uniform by $o(1)$ in total variation distance.
CODES FROM COMPLETE BIPARTITE GRAPHS AND 3-REGULAR GRAPHS

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(Joint work with Keisuke Shiromoto)

Recently, there has been a lot of researches on graphical codes, that is, linear codes obtained from graphs. In particular, the binary code generated by a complete graph $K_n$ is well known. In 1997, Jungnickel and Vanstone proved that this code is contained in the Hamming code of length $2^n - 1$ if and only if $n$ is one of the number 2, 3 and 6. Then it is natural to study the problem when the binary code generated by a complete bipartite graph or a 3-regular graph is contained in some binary Hamming code. In this talk, we shall explain some results concerning the problem.

BARNETTE’S CONJECTURE: FINDING INDUCED TREES

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(Joint work with Charl Ras [Supervisor])

Barnette’s conjecture states that every planar, cubic, 3-connected, bipartite graphs is Hamiltonian. We shall refer to these graphs as Barnette graphs, and denote them by $B$. We shall approach this conjecture by considering the duals of the graphs in $B$, and then converting the conjecture into another $NP$-complete problem, namely, that of finding induced subtrees of graphs. We will first discuss various properties of these induced trees and then prove new necessary conditions for finding Hamiltonian Cycles in graphs of $B$. 
RAINBOW CYCLES IN EDGE-COLORED GRAPHS
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Let $G$ be a graph of order $n$ with an edge coloring $c$, and let $\delta^c$ denote the minimum color degree of $G$, i.e., the largest integer such that each vertex of $G$ is incident with at least $\delta^c$ edges having pairwise distinct colors. A subgraph $F \subset G$ is rainbow if all edges of $F$ have pairwise distinct colors. In this talk, we introduce our result that (i) if $\delta^c > n/2 + 2$, then $G$ contains a rainbow cycle of length at least 4, and (ii) if $G$ is triangle-free and $\delta^c > n/3 + 1$, then $G$ contains a rainbow $C_4$.

CONSTRUCTIONS OF CHIRAL POLYTOPES
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(Joint work with Marston Conder)

Abstract polytopes are combinatorial structures with certain properties drawn from the study of geometric structures. An abstract polytope is called regular if it has maximum possible symmetry (including reflections). Chiral polytopes are abstract polytopes which have maximal symmetry by rotation, but admit no reflection. Examples of chiral polytopes have been difficult to find and construct. In 2005, Conder, Hubard and Pisanski identified the smallest examples of chiral polytopes of ranks (dimensions) 3 and 4, and found the first known examples of finite chiral polytopes of rank 5 (now known to be the smallest). The existence of chiral polytopes of each rank greater than 2 was proved by Daniel Pellicer in 2010, but his examples were extremely large and somewhat difficult to understand. In this talk, we will introduce some new methods of constructing chiral polytopes. In particular, we will describe the identification of the smallest chiral polytopes of rank 6. This is joint work with my supervisor Marston Conder.
HADWIGER’S CONJECTURE FOR THE COMPLEMENTS OF KNESER GRAPHS

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(Joint work with Guangjun Xu, Zunyi Normal College and the University of Melbourne)

Hadwiger’s conjecture asserts that every graph with chromatic number $t$ contains a complete minor of order $t$. A stronger conjecture due to Hajós asserts that every graph with chromatic number $t$ contains a subdivision of $K_t$. Hajós’ conjecture fails for every $t \geq 7$, and if Hadwiger’s conjecture is false then counterexamples must be found among counterexamples to Hajós’ conjecture. In 2005, Thomassen presented several new classes of counterexamples to Hajós’ conjecture, including the complements of the Kneser graphs $K(3k-1, k)$ for sufficiently large $k$. (Given $n \geq 2k+1 \geq 5$, the Kneser graph $K(n,k)$ is the graph with vertices the $k$-subsets of an $n$-set such that two vertices are adjacent if and only if the corresponding $k$-subsets are disjoint.)

We prove that Hadwiger’s conjecture is true for the complements of Kneser graphs. In the case when $2k+1 \leq n \leq 3k-1$, the independence number of the complement of $K(n,k)$ is equal to 2, and we show that the gap between the Hadwiger number and the chromatic number can be arbitrarily large when $n$ and $k$ vary. In general, Hadwiger’s conjecture for graphs of independence number 2 is an interesting but challenging problem.